**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

Ans: mean, µ = 45, Std dev, *σ = 8*

Work after 10 min and work should be completed in I hour. So, the working time required is, x = 50

Probability that works gets completed within 50 min is P(X<=50).

Probability that the manager cannot meet the commitment is P(X>50) = 1-P(X<=50)

P(X<=50) = stats.norm.cdf(50, loc=45, scale=8)

= 0.7341

P(X<=50 is 73.41%

P(x>50) = 1- stats.norm.cdf(x=50, loc=45, scale=8)

= 0.26598552904870054

P(X>50) is 26.59%

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:

Population = 400

mean = 38

std dev =6

P(X>44) and P(38<X<44)

P(X<=44) is 84.13%

P(X>44) => 1 - Stats.norm.cdf(44, 38, 6) is 15.86%

And P(38<X<44)=> Stats.norm.cdf(44, 38, 6) – stats.norm.cdf(38,38,6) is 34.13%

Since, P(38<X<44) > P(X>44), the statement given is FALSE, meaning that the population between age 38 and 44 is more than the population with age greater than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

P(X<30) = stats.norm.cdf(30,38,6) is 9.12%

no\_of\_employees\_below\_age\_30 = 0.0912\*400 = 36.48 which approximately 36 employees.

Hence, the statement is TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As iid means independent and identically distributed normal random variables,

1. Distribution of 2X1:

If we multiply a normal random variable X1 by a constant, the mean gets scaled by that constant, and the variance gets scaled by the square of that constant.

So, 2X1 ~ N(2μ, 4σ2)

2. Distribution of X1 + X2:

The sum of two independent normal random variables is also a normal random variable. The mean of the sum is the sum of the means, and the variance of the sum is the sum of the variances.

Therefore, X1 + X2 ~ N(2μ, 2σ2)

Both random variables have the same mean (2μ), but the variance differs: 2X1 has a larger variance (4σ²) compared to X1 + X2 (2σ²). This is because multiplication by a constant increases the spread of the distribution.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

Ans: mean = 100, SD =20

Probability of any random variable between a and b is 0.99.

So, the probability of random variable being outside area is 1-0.99 = 0.01

Alpha is 0.01

The Probability towards left from a is 0.01/2 = 0.005

The Probability towards right from b is 0.01/2 = 0.005

Z = (X-mean)/std

Z-score of 0.005 is = -2.57

Z\*std + mean = X

Confidence interval = 100 +/- 20\*2.57

= (48.6, 151.6)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: sum of two independent division distribution profit = profit1 + profit2

= N(5+7, 3^2+4^2) = N(12, 5^2)

Mean is 12\*45 = 540 million

SD is 5 \* 45 = 225 million

Interval is (99008103.47848785, 980991896.5215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: to find the 5th percentile value of profit, find Z alpha

Z alpha 0.05 = -1.644584

Therefore, 5th percentile value of profit = (-1.644584\*5)+12 = 3.777

In rupees 3.777\*45 = 170 million rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: loss probability = 1 – profit probability

= 1-stats.norm.cdf()

Probability of first company not making any profit = 0

(if there is no profit, there are in loss)

Not making profit , X= 0

Profit1 ~ N(5,3^2)

Mean = 5, std = 3

Z = X-mean/std = (0-5)/3

Z = -1.667

Profit2 ~N(7,4^2)

Mean =7, std =4

Z = (X-mean)/std

= (0-7)/4

=-1.75

Thus the probability that the company making more loss is second division in the company.